Identical Quantum Particles and Weak Discernibility

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Abstract

We examine a recent argument that "identical" quantum particles with an anti-symmetric state (fermions) are *weakly discernible* objects, just like irreflexively related ordinary objects in situations with perfect symmetry (Black's spheres, for example). We conclude that the argument uses a silent premise that is not justified in the quantum case.

1   Individuality

Objects are individuals. One might think of their individuality as something primitive, or as grounded in a metaphysical principle not accessible to science—but here we shall not consider such haecceistic conceptions. We take as our starting point that entities must differ in their qualitative features, described by the relevant scientific theory, in order to be treated as different objects. This is in the spirit of Leibniz's principle of the identity of indiscernibles (PII).

At first sight it may seem that PII cannot function as a general basis for individuality, since it does not do justice to cases with several objects in a symmetrical configuration. Think of Max Black's spheres, of identical composition and two miles apart in a relational space (à la Leibniz, not in Newtonian absolute space where absolute positions could label the spheres); Kant's enantiomorphic hands; or, for a mathematical example, the points in

the Euclidean plane. In all these cases there are obviously more than one individual objects, but it also appears clear that these objects share all their qualitative features. Both spheres in Black's example have the same material properties and both are at two miles distance from a sphere; similarly, Kant's hands have the same internal geometric properties and are both mirror images of a hand; and so on. However, the impression that PII is in trouble here is superficial. As emphasized by Saunders [5, 6], who takes his clue from Quine [3], it is essential to note that in these examples *irreflexive* qualitative relations exist between the entities—relations an entity cannot have to itself. This irreflexivity is sufficient to prove that PII is satisfied after all.

There are some subtleties here that deserve attention. To discuss these, let us first formalize the above argument. In formal first-order languages we can *define* identity (=):

$$s = t \equiv P(s) \leftrightarrow P(t), \tag{1}$$

where P denotes an arbitrary predicate in the language, and the right-hand side of the definition stipulates that s and t can replace each other, *salva veritate*, in any P. This definition captures PII, and the above discussion of individuality, if the language is that of a physical theory, in which the predicates refer to physical properties and relations (and not to haecceities).

There can now be three kinds of discernibility ([6]). Two objects are *absolutely discernible* if there is a one-place predicate that applies to only one of them; *relatively discernible* if there is a two-place predicate that applies to them in only one order; and *weakly discernible* if an *irreflexive* two-place predicate relates them. The latter possibility is relevant for our examples. If there is an irreflexive but symmetric two-place predicate $P(.,.)$ satisfied by s and t, the definition (1) requires that if s and t are to be identical, we must have:

$$\forall x (P(s, x) \leftrightarrow P(t, x)). \tag{2}$$

But this is false: in a valuation in which $P(s, t)$ is true, $P(t, t)$ cannot be satisfied by virtue of the fact that P is irreflexive. Therefore, it immediately follows that PII is satisfied by any two objects that are irreflexively related. The only thing needed to dispel the impression that PII is in trouble in these cases is therefore to apply the principle not only to monadic predicates, but also to relations.

There might seem to be a remaining difficulty. In the above argument we made use of the notion of a *valuation*. A valuation results from letting the

names and bound variables in the formulas of the language refer to specific elements of the intended domain. In other words, in order to construct a valuation, we have to name and distinguish the things we are discussing. But this is an impossible task in the symmetrical configurations we have been considering. Because of the symmetry, any feature that can be attributed to one element of the domain can also be attributed to any other. We can therefore not uniquely refer and assign names on the basis of the given structure of properties and relations. It is clearly impossible, for example, to single out any *specific* point in the Euclidean plane on the basis of the properties of the plane and its points, even if we include all relational properties. This impossibility might appear to take the edge off the above argument; and it might be thought to threaten the individuality of our entities after all (because individuals can bear names, whereas no names can be given here) [2].

The difficulty is only apparent, however. In order for the notions of number and individuality to apply to the members of a domain it is sufficient that a function *exists* that maps the domain one-to-one onto a set of labels, e.g. the set $\{1, 2, ..., n\}$ [8, p. 457]. It is not needed that we can actually *construct* such a labelling. In all the cases we have been considering the required mappings exist, and provide us with valuations.

Although the name-giving problem is thus dissolved, it has drawn our attention to an essential aspect of our examples of weak discernibility: in all these cases the domains possessed a structure that underwrote the existence of bijections to sets of labels (e.g., sets of one or more natural numbers). As we shall illustrate in a moment, the mere possibility of speaking about a domain in terms of irreflexive relations is not enough to ensure such an "object structure": it is possible to use properties or relations talk even in situations in which there are no objects at all. In such cases arguments about irreflexivity and weakly discernible individuals obviously cannot come off the ground. This means that we have to decide whether the concept of an object applies in the first place, before we can use irreflexivity of the relations to prove that the objects in question are weakly discernible individuals.

This does not necessarily imply that there must be a division of labor between "objecthood providers" and the irreflexive relations. The relational structure may be the only structure of the domain that is available and may provide our only access to objecthood. The relations in our two physical examples—being at a spatial distance from each other, being each other's mirror image—indeed give us information about the presence of objects. They are defined for relata that are physical things: things that can be

displaced with respect to each other, that can be reflected and whose orientations can be compared. Such relations apply to objects; conversely, if such relations apply, there are one or more objects as relata. Objects are able to enter into more relations, and this provides us with an additional test for objecthood that applies to the cases mentioned thus far. Indeed, why are we so sure that there are two Blackian spheres and two Kantian hands? Our mind's eye sees Black's spheres at different distances, and Kant's hands with different orientations, before us. Speaking generally, when we break the symmetry of a configuration of objects by introducing a reference object, gauge or standard, the objects will become distinguishable with respect to this standard. Thus we can name Black's spheres via their unequal distances to a reference sphere and in Kant's universe we may imagine a reference hand conventionally called "left". Another example, relevant for our subsequent discussion of quantum objects, is furnished by two oppositely directed arrows in an otherwise empty Leibnizean world. If we conventionally fix a standard of being "up", we break the symmetry and the individual arrows become up or down—which verifies that they were weakly discernible *objects* in the original setting.

As already announced, it is essential for our subsequent argument that irreflexive relations do not automatically connect objects, and do not always pass the just-mentioned test. Consider the example of Euros in a bank account (not coins in a piggy bank, but transferable money in a real bank account). Imagine a situation in which by virtue of some financial regulation the Euros in a particular account can only be transferred to different one-Euro accounts. So, in a complete money transfer an account with five Euros, say, would be emptied and five different one-Euro accounts would result. In this case the Euros in the original account stand in an irreflexive relation to each other, namely "only transferable to different accounts". But this does not make them into physical objects. Analogously to the earlier cases, we could try to exploit the irreflexive relations by introducing an additional standard Euro that can only be transferred to one specific account; or more directly, we may attempt to label the Euros by means of the accounts they can end up in. However, this does not achieve anything for the purpose of distinguishing between the Euros in the account they are actually in. The essential point is that the relations here do not pertain to occurrent, actual, physical features of the situation; they do not connect actual physical relata. Rather, the relational structure is defined with respect to what *would* result if the actual situation *were changed*. There is of course no doubt that five different one-

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Euro accounts (with different account numbers) are five individuals; but this does not mean that it makes sense to consider the five Euros as individual objects before the transfer, when they are still in one common account. On the contrary, the case of more than one money units in one bank account is the standard example of classical non-individuals, where only the account itself, with the total amount of money in it, can be treated as possessing individuality [4, 7]. Although we may use relations and things *talk* here, there is nothing in the actual physical situation that directly corresponds to this. Speaking of several Euros in a bank account is a *façon de parler* and does not refer to actual physical objects. According to our best theoretical understanding of the situation, statements like "all Euros in this account have the same value—namely one Euro" are not about actual physical things.

Summing up, we have found an important silent presupposition in the argument for PII-based individuality in the presence of irreflexive relations. Such relations can only be trusted to be significant for the individuality issue if they are of the sort to connect actual relata, objects. One way of verifying this is to look into the relations' meaning and role in the pertinent scientific theory. Another is to make use of a consequence of objecthood: breaking of the perfect symmetry will result in the possibility of giving names. This leads to a necessary condition, a test: break the symmetry and see if names become assignable. The breaking of the symmetry is analogous to the introduction of a coordinates origin in describing a figure in plane geometry. If a mapping to natural numbers exists in the presence of such a reference point, it will still be there when the reference point has been removed; what changes is the *constructibility* of the mapping. For the *existence* of a mapping it is only relevant whether the relations are such that they link actual things; the introduction of an external vantage point makes it possible to assign names, but does not change objecthood or the number of objects.

2 The Quantum Case

One notorious interpretational problem of quantum theory concerns so-called identical particles: particles of the same kind, like electrons, protons or neutrons. It is a principle of quantum theory that the state of a collection of such particles is completely symmetric (in the case of bosons) or anti-symmetric (fermions). This symmetrization postulate implies that all one-particle states occur symmetrically in the total state of a collection of identical particles. It

follows that any property or relation that can be attributed, on the basis of the total quantum state, to one particle is attributable to all others as well.

The standard response to this conclusion is to say that identical quantum particles lack individuality. This is tantamount to saying that as far as the physical description goes they are no particles at all: there may be "many of them", but this is like many Euros in a bank account. It is better, according to this received view, to renounce talk that suggests the existence of individual particles—we should reconceptualize the situation in terms of the excited states of a field (analogous to thinking of the Euros in an account as one sum of money).

However, the situation also reminds us of the symmetric classical cases described in the previous section. As we have seen there, symmetry is not enough to prove the absence of non-haecceistic individuality: particles may still be weakly discernible individuals. Could it not be that in the quantum case there are irreflexive physical relations between particles that guarantee their individuality in the same way as they did for Black's spheres, Kant's hands and Euclid's points? This is the position adopted by Saunders, at least for the case of fermions [6]. Indeed, the anti-symmetry of the state of many-fermions systems implies the existence of irreflexive relations: intuitively speaking, the fermions in any pair stand in the relation of "occupying different one-particle states", even though any specific state description applicable to one of them applies equally to the other. It is true (and noted by Saunders) that for bosons with their symmetrical states this manoeuvre is not available, so that collections of identical bosons are still best understood as one whole. But the conclusion that standard quantum mechanics[1] entails that fermions (these are the ordinary matter particles; bosons are quanta of interaction fields) are ordinary individuals—though only weakly indiscernible—is surprising and highly significant by itself.

The technical details of the argument can be illustrated by the example of two fermions in the singlet state. If $|\uparrow\rangle$ and $|\downarrow\rangle$ stand for states with spins directed upwards and downwards in a particular direction, respectively, the anti-symmetrization principle requires that a typical two-fermion state looks

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[1]It is simple to promote quantum particles to the status of unproblematic individuals by *adding* individuating characteristics to the descriptions given by the theory, as is done, e.g., in the Bohm interpretation. The problem is that these added features are empirically superfluous—anyway, here we look only at standard quantum mechanics, like Saunders does.

like

$$\frac{1}{\sqrt{2}}\{|\uparrow\rangle_1|\downarrow\rangle_2 - |\downarrow\rangle_1|\uparrow\rangle_2\}, \tag{3}$$

in which the subscripts 1 and 2 refer to the one-particle state-spaces of which the total state-space (a Hilbert space) is the tensor product. These two one-particle state-spaces are the available candidates for the description of single particles, so their labels are candidate names for the individual fermions (we are assuming hypothetically that this notion makes sense). Now, the anti-symmetry of the total state implies that the state restricted to state-space 1 is the same as the restricted state defined in state-space 2. (The "partial traces" are $\frac{1}{2}\{|\uparrow\rangle\langle\uparrow| + |\downarrow\rangle\langle\downarrow|\}$ in both cases.) The *total spin* has the definite value 0 in state (3); that is, state (3) is an eigenstate of the operator $S_1 \otimes I + I \otimes S_2$. Therefore, it seems natural to say that the two spins are *oppositely directed*. On the other hand, we cannot assign a definite spin direction to the single particles because the up and down states occur completely symmetrically in each of the Hilbert spaces 1 and 2, respectively.

This situation is reminiscent of that of weakly discernible classical objects; in particular it appears essentially the same as the case of the two arrows. As we have seen in the latter case, it is not possible to designate one of the arrows as up and the other as down—but nevertheless there must be two individual arrows in view of the oppositeness of their directions. Similarly, in the fermion case with total spin zero we appear to have two individual quantum objects with opposite spins.

3   Quantum Individuals?

On closer examination the similarity starts to fade away, however. One should already become wary by the observation that the irreflexive relations in the quantum case have a formal representation that is quite different from that of their classical counterparts in the cases we contemplated earlier. There the relations could be formalized by ordinary predicates that can be expressed as *functions* of occurrent properties of the individual objects (like "up" and "down", or $+1$ and -1, with the correlation expressed by the fact that the sum of these two quantities vanishes). By contrast, here we have that the total system is in an *eigenstate* of a linear *operator* in the total system's Hilbert space. Concomitant with this formal difference is an essential difference in interpretation: according to standard quantum mechanics, quantum

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states should be interpreted in terms of possible *measurement results* and their probabilities, rather than in terms of occurrent properties. In the case at hand, a system in an eigenstate of the total spin operator with eigenvalue 0, this means that a measurement of the total spin will have the outcome 0 with probability 1. In this special case, in which the outcome is certain (probability 1), it is plausible and harmless to think that the total system possesses the property "total spin 0" also independently of measurement; but even if we assume this, this total spin cannot be understood as being composed of definite spin values of the two subsystems. Although it is of course possible to perform individual spin *measurements* on the subsystems (whose possible outcomes and corresponding probabilities are predictable from the total quantum state), there are no corresponding *occurrent spin properties* in the subsystems, independently of measurement. In the singlet state (3) the prediction of quantum mechanics is that individual spin *measurements* will with certainty yield opposite results, summing up to 0; but on the pain of running into paradoxes and no-go theorems it cannot be maintained that these results reveal oppositely directed spins that already were there before the measurements. This is an example of the notorious "holism" of quantum mechanics: definite properties of a composite system need not be reducible to properties of its parts.

This suggests that the correct analogue to the quantum case is not provided by two oppositely directed classical arrows, but rather by a two-Euro account that can be *transformed* (upon "measurement", i.e. the *intervention* brought about by a money transfer) into two distinct one-Euro-accounts. To investigate further whether or not the quantum relations connect actual physical relata, we may copy the strategy followed in the classical case, namely breaking the symmetry and seeing whether in the resulting situation the quantum relations can serve as name-givers. This cannot work as long as we stay within a many-fermions system: quantum mechanics forbids fermion systems that are not in an anti-symmetric state—accordingly, it is a matter of lawlike principle that the only relations fermions can possess with respect to each other are perfectly symmetrical. So if we want to break the symmetry this should be done by the introduction of a standard that is external to the fermion system. After this we are in a position to verify whether the quantum relations with respect to the introduced standard make the individual fermions distinguishable.

To see the inevitability of a negative outcome of any such attempt, consider an arbitrary system of identical quantum particles to which a gauge

system has been added without any disturbance (i.e., the total state is the product of the original symmetrical or anti-symmetrical identical particles state and the state of the gauge system). Let the new total state be denoted by $|\Psi\rangle$. Any quantum relation in this state between the gauge system and one of the identical particles, described in subspace j, say, has the form $\langle\Psi|A(g,j)|\Psi\rangle$. Here $A(g,j)$ is a hermitian operator working in the state-spaces of the gauge system g and identical particle j. We can now use the (anti)-symmetry of the original identical particles state to show that the gauge system stands in exactly the same relations to *all* identical particles. The (anti)-symmetry entails that $P_{ij}|\Psi\rangle = \pm|\Psi\rangle$, where P_{ij} stands for the operator that permutes indices i and j. Now,

$$\langle\Psi|A(g,j)|\Psi\rangle = \langle P_{ij}\Psi|A(g,j)|P_{ij}\Psi\rangle =$$
$$\langle\Psi|P_{ij}^{-1}A(g,j)P_{ij}|\Psi\rangle = \langle\Psi|A(g,i)|\Psi\rangle.$$

In other words, any quantum relation the gauge system has to j, it also has to i, for arbitrary values of i and j. That means that these quantum relations have no discriminating value in the situation as it actually is, without measurement interventions and the disturbances caused by them.

It must be stressed that if the situation *is changed* by a measurement interaction, distinct individual results may arise[2], just like in the case of the opposite spin results in measurements in the total spin 0 state (3). But we are here interested in the question of whether a many-fermions system *as it is* can be regarded as a collection of weakly discernible individuals; not in the question of whether such a system can be *transformed* into a collection of individuals. The fermions behave like money units in a bank account: it does not matter what external standard we introduce, it will always possess the same relations to all (hypothetically present) entities. This leaves us without evidence that there *are* any actual objects composing many-fermions systems. This stands in sharp contrast to our earlier cases of classical weakly discernible objects.

4   Conclusion

There is an essential difference between quantum mechanical many-fermion systems and classical collections of weakly discernible objects. In the latter

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[2]The expressions $\langle\Psi|A(g,j)|\Psi\rangle$ are in this case interpreted as *expectation values*, averages over very many experimental trials.

case the objects are nameable *in abstracto*, although the symmetry of the situation makes it impossible to actually assign names. This is a strange situation; but the air of paradox is dispelled when we apply the concept of weak discernibility. The strangeness of the quantum case runs much deeper. There is no sign within standard quantum mechanics that identical fermions are things at all; the irreflexivity of relations does not help us here. Quantum relations have an interpretation not in terms of what is actual, but rather via what *could* happen in case of a measurement; and they cannot be used in a name-giving procedure after the introduction of an external standard. There is therefore no evidence that the quantum relations between fermions connect any actual physical objects. As far as standard quantum mechanics goes, identical fermions are not discernible, not even weakly. Conventional wisdom, saying that systems of identical quantum particles should be considered as one whole, like an amount of money in a bank account, appears to be right after all.

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